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Study of Submarine Buoys Moored by Two Flexible Cables

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Many underwater oceanographic instruments are moored with cables. An analysis to determine the steady-state configuration of an underwater-suspended bipod cable system is presented. A method developed by the author which allows the computation of shape and tensions of a tridimensional, weighted, extensible, mooring line in a nonuniform stream is recalled. Polynomial laws are used to represent stream velocity and direction variations vs depth. An iterative solution is developed for the equilibrium configuration of the bipod buoy system. The simplicity of this method permits use of a microcomputer. An example of complete application is presented.

Introduction

MANY underwater-suspended bipod cable systems are used to moor oceanographic instrumentation. An analysis to determine the steady-state configuration of the system subject to buoyant lift forces as well as current drag forces is necessary. A method for computing the displacements, due to currents, of a submarine buoy moored by two cables was presented by Mariano.[1] His assumptions, however, were too restrictive. We are now suggesting our own method for solving this problem in a more general manner, which seems to be easier to implement.

Mariano's Work Analysis

The system analyzed comprises two identical flexible cables with their lower ends separated and anchored on the ocean bottom (in a horizontal plane), and their upper ends joined together and attached to a subsurface buoy, as shown in Fig. 1.

Mariano's intention was to determine the buoy position and the cable tensions given any current strength and direction in the mooring site. To begin with, he assumes that the cable has no apparent weight and he neglects the tangential hydrodynamic forces. In these conditions, he considers that any cable element is only subjected to normal forces due to pressure. It follows that, for any velocity field inside the current or for any cable shape, the tension is a constant in any point, which is, of course, unrealistic. Furthermore, he assumes that the current velocity remains in any point parallel to a fixed direction. It is then true to say that the equilibrium shape of a weightless cable in plane (the cable is in a plane defined by the directions of the force at one end and of the current). But this is not the general case. Finally, Mariano computes the local speed component normal to the cable as if it were a straight line. This is a very misleading approximation when the cable curvature is important.

In spite of these assumptions, the proposed solution seems to be rather complicated.

General Solution of the Tridimensional Equilibrium of a Submarine Cable

Let us briefly recall the hypothesis we used and the equations we obtained to find the tridimensional shape and the tensions of cables in equilibrium in any current. The complete mathematical details of this theory can be found in Refs. 2-4.

Current Description (Fig. 2)

Assume that the cable is subjected to a current of depth H, the velocity of which is horizontal everywhere, but whose direction and intensity are described by two series as:

$$V = V_M \sum_{i=0}^{n} a_i \left(\frac{Z}{H}\right)^i \tag{1}$$

and

$$\varphi = \sum_{i=0}^{n} b_i \left(\frac{Z}{H}\right)^i \tag{2}$$

where V is the velocity modulus and φ the angle the velocity vector makes with the x axis. Measurements at the mooring site, or some hypothesis, can give the a_i and b_i coefficients.

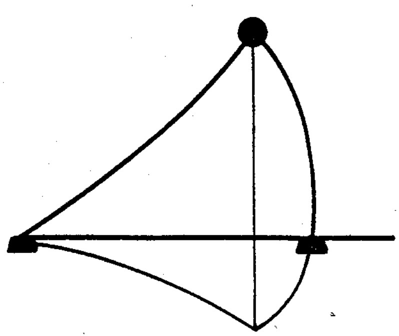

Fig. 1 Cable configuration.

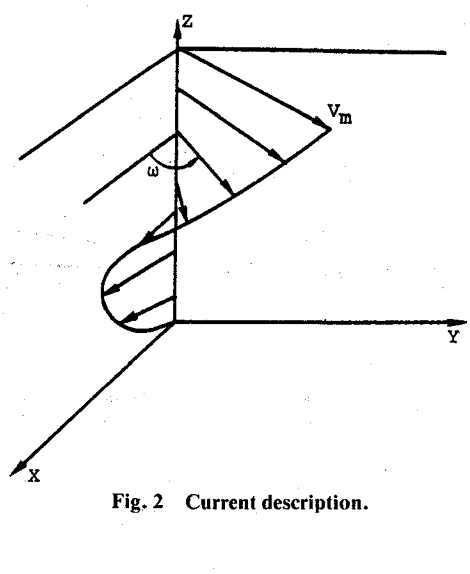

Fig. 2 Current description.

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Systems of Coordinates (Fig. 3)

The absolute coordinate system $OXYZ$ is defined as:

1) The origin O is one cable end. For convenience, we generally choose the cable end where the tension is known (if possible, in both magnitude and direction).
2) The X axis is a characteristic direction of the problem.
3) The Z axis is upward vertical.
4) The Y axis is chosen so that the $OXYZ$ system is direct.

The $Oxyz$ system is related to the flow velocity direction at any point along the cable:

1) The x axis is the local velocity direction and orientation.
2) The origin O is the same as the absolute system one.
3) The z axis is upward vertical (the same as OZ).
4) The y axis is so that the $Oxyz$ system is direct.

The $Oxyz$ system is obtained from the $OXYZ$ system through a rotation of angle φ about the Z axis.

The $Mtnb$ system is related to any cable element. With M as the origin, the t axis is tangent to the cable and oriented from the origin to the end of the cable.

Let $\mathcal{P}$ be the vertical plane containing the Mt axis; this plane crosses the horizontal plane xOy along the $O'P$ axis; the $O'z'$ axis is upward vertical.

The θ and ψ angles are then defined by:

$$\theta = (Ox,\ O'P) \qquad \psi = (O'P,\ Mt)$$

The first normal to the cable, Mn, is perpendicular to the plane and is such that the $O'P$, Mn, $O'z'$ trihedral is direct. The second normal, Mb, is such that the $Mtnb$ system is also direct.

Forces Acting on the Cable

We extended Landweber's hypothesis[5] to tridimensional cables:

1) The cable is round and smooth.
2) It is infinitely flexible.
3) It is extensible. However, the elongations have to remain small enough to neglect the section deformation.
4) The local curvature of the cable does not affect the hydrodynamic forces acting on each element.
5) The hydrodynamic forces acting on each cable element, the length of which is ds, are made of:

a) The normal force due to pressure strength. If V_l is the velocity V projected on the plane perpendicular to the cable in the considered point (Mnb plane), the normal force has the following components in the $Mtnb$ system:

$$R_n = \tfrac{1}{2}\rho C_D D V_l^2 \cos\alpha\, ds \tag{3}$$

$$R_b = \tfrac{1}{2}\rho C_D D V_l^2 \sin\alpha\, ds \tag{4}$$

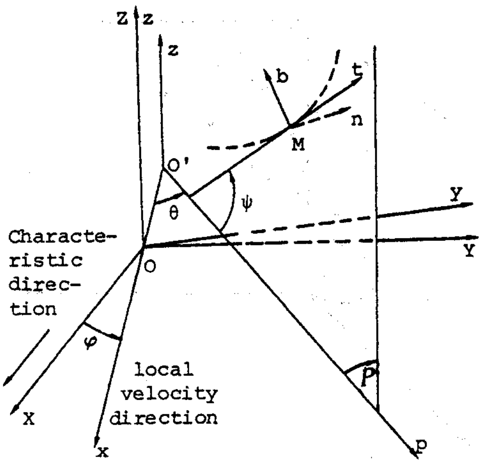

Fig. 3 Systems of coordinates.

where C_D is the drag coefficient of a cable element submitted to a normal current; D the cable diameter; and α the angle between V_l and Mn.

b) The friction-trangential force, independent of the cable inclination, which can be written as:

$$R_t = f\frac{1}{2}\rho C_D D V \frac{|V\cos\theta\cos\psi|}{\cos\theta\cos\psi}\, ds \tag{5}$$

where f is a constant friction coefficient and where the $|V\cos\theta\cos\psi|/\cos\theta\cos\psi$ term takes into account the fact that the friction force cannot have a negative component along the current direction.

Equilibrium Equations

Let us now study the equilibrium of a cable element, the length of which is ds. It is submitted to the following hydrodynamic forces, its apparent weight, $W ds$, and some tensions exerted by the contiguous elements.

Let θ and ψ be the angles defining the tangent to the element at one of its ends, T the cable tension at that end, $\Delta\theta$, $\Delta\psi$, and ΔT their changes, respectively, remaining small by hypothesis along the element. This gives the scheme presented in Fig. 4.

The hydrodynamic forces are computed in the middle of the segment ds, that is, for $\theta + (\Delta\theta/2)$ and $\psi + (\Delta\psi/2)$. The equilibrium equation is then obtained in writing, under a linearized form if necessary, that the force components along each n, b, and t axis have a zero resultant.

Defining,

$$P = \sqrt{\sin^2\theta + \cos^2\theta\sin^2\psi} \tag{6}$$

$$\tau = T/R ds \tag{7}$$

$$w = W/R ds \tag{8}$$

with R the drag per unit length of a straight cable element perpendicular to a uniform current whose strength is the one of the local flow ($R = \tfrac{1}{2}\rho C_D D V^2$).

n axis:

$$\frac{\Delta\theta}{2}\left[2\tau\cos\psi - P\cos\theta\left(1 + \frac{\cos^2\psi\sin^2\theta}{P^2}\right)\right] - \frac{\Delta\psi}{2}\left[\frac{\cos^2\theta\sin\theta\sin\psi\cos\psi}{P}\right] = P\sin\theta \tag{9}$$

b axis:

$$\frac{\Delta\theta}{2}\left[P\sin\theta\sin\psi\left(1 - \frac{\cos^2\psi\cos^2\theta}{P^2}\right)\right] + \frac{\Delta\psi}{2}\left[2\tau + w\sin\psi - P\cos\psi\cos\theta\left(1 + \frac{\cos^2\theta\sin^2\psi}{P^2}\right)\right] = w\cos\psi + P\cos\theta\sin\psi \tag{10}$$

t axis:

$$\Delta\tau = w\sin\left(\psi + \frac{\Delta\psi}{2}\right) - f\frac{V\cos\theta\cos\psi}{|V\cos\theta\cos\psi|} \tag{11}$$

We can add to these equilibrium equations the geometrical relations:

$$dx = ds\left[\cos\left(\theta + \frac{\Delta\theta}{2}\right)\cos\left(\psi + \frac{\Delta\psi}{2}\right)\cos\phi - \sin\left(\theta + \frac{\Delta\theta}{2}\right)\cos\left(\psi + \frac{\Delta\psi}{2}\right)\sin\phi\right] \tag{12}$$

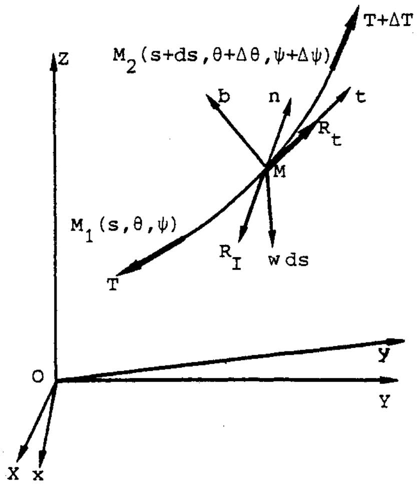

Fig. 4 Cable element equilibrium.

$$dY = ds\left[\cos\left(\theta+\frac{\Delta\theta}{2}\right)\cos\left(\psi+\frac{\Delta\psi}{2}\right)\sin\phi + \sin\left(\theta+\frac{\Delta\theta}{2}\right)\cos\left(\psi+\frac{\Delta\psi}{2}\right)\cos\phi\right] \tag{13}$$

$$dZ = ds\sin\left(\psi+\frac{\Delta\psi}{2}\right) \tag{14}$$

Solution

We can see that Eqs. (9) and (10) give a 2×2 linear system in which $\Delta\theta$ and $\Delta\psi$ are unknowns and where ΔT does not interfere. This peculiar form of the equations suggests a step by step solution method which gives the shape and tension characteristics of the entire cable.

In fact, the force applied to one end of the cable is generally known (or one can impose it). T, ψ, and θ are, therefore, known for the first element. Applying Eqs. (9) and (10) on this element in choosing its length ds gives the $\Delta\theta$ and $\Delta\psi$ changes. All equilibrium characteristics of the first element are then determined from Eqs. (11-14). In the same way, each element up the cable extremity can be analyzed successively (an appropriate test suggested by the problem stops the computing procedure).

In the case of an inextensible cable, it is easier to keep the length of each element a constant. One can generally use an entire fraction of any problem characteristic (cable length, depth, etc.). For extensible cables, one can select any reference length ds_0 just as in the former case, any cable element having a changing length given by:

$$ds = ds_0\left(1+\frac{T}{ES}\right) \tag{15}$$

where E is Young's modulus of the material and S the cable section.

Many examples have been treated in the aforementioned publications showing the accuracy of our method.

Method Applied to a "Bipod" Mooring System

A spherical submarine buoy is moored with two cables. Its buoyancy N and its drag are known. The cable mechanical characteristics (apparent weight, length, drag coefficient, etc.) are also given. Let us assume in this example (but it is not necessary), that the cables are inextensible.

The lower ends are fixed in the same horizontal plane. This hypothesis is just used to solve the same problem as Mariano. Let Δ be the distance between the mooring points.

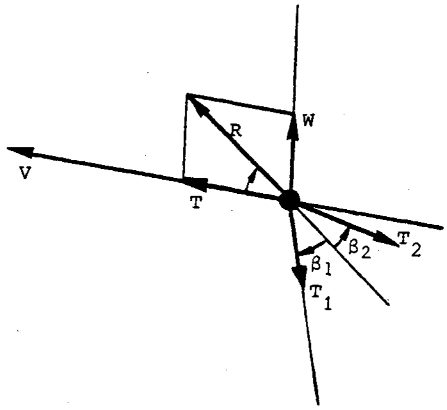

Fig. 5 Buoy equilibrium.

The absolute $OXYZ$ system is chosen as follows: the origin O is in the middle of the mooring points; the OY axis passes through these points; the OZ axis is upward vertical; the OX axis is such that the trihedral is direct.

In these conditions, the "current angle" is the angle between the velocity direction and the mediating plane of the mooring segment.

Both cables are distinguished by means of subscripts *1* and *2*. The cable tensions in the mooring points are completely unknown. Writing any buoy equilibrium, the resolution is initialized with some tensions in the upper cable elements.

Buoy Equilibrium

The buoy size is generally small compared to the cable length. Therefore, the buoy will be considered as a material point where its buoyancy, drag, and the link forces with the cables are concentrated.

Since the buoy is spherical, the drag force is parallel to the local flow velocity. Combining drag and buoyancy forces, the resulting force R acting on the buoy is obtained. (R lies in the vertical plane containing the buoy and the local flow velocity direction. α is the angle between R and this direction.) The buoy is, therefore, submitted to three forces: R and the cable tensions T_1 and T_2 (Fig. 5). In order to be in equilibrium, these forces have to be in the same plane, but the position of this plane is a priori unkown.

Let γ define the angle between oy and oy', the perpendicular to R in the T_1, T_2 plane. T_1 and T_2 stand at angles β_1 and β_2, respectively, to the direction of R. The buoy equilibrium equations are then simply written as:

$$T_1\cos\beta_1 + T_2\cos\beta_2 = R \tag{16}$$

$$T_1\sin\beta_1 + T_2\sin\beta_2 = 0 \tag{17}$$

It is now necessary to find the relationships between the angles α, β_1, β_2, and γ and the angles ψ_1, ψ_2, θ_1, and θ_2, which define the tension directions in the cable elements attached to the buoy (Fig. 6).

The usual matrix relationships, corresponding to each formerly defined rotation, are used to establish that the tension components in the $Oxyz$ system are given by:

$$T_{x_1} = (\cos\alpha\cos\beta_1 + \sin\gamma\sin\alpha\sin\beta_1)\,T_1 \tag{18}$$

$$T_{y_1} = \cos\gamma\sin\beta_1\,T_1 \tag{19}$$

$$T_{z_1} = (-\sin\alpha\sin\beta_1 + \sin\gamma\cos\alpha\sin\beta_1)\,T_1 \tag{20}$$

Then, for the first cable, we get:

$$tg\theta_1 = T_{y_1}/T_{x_1} \tag{21}$$

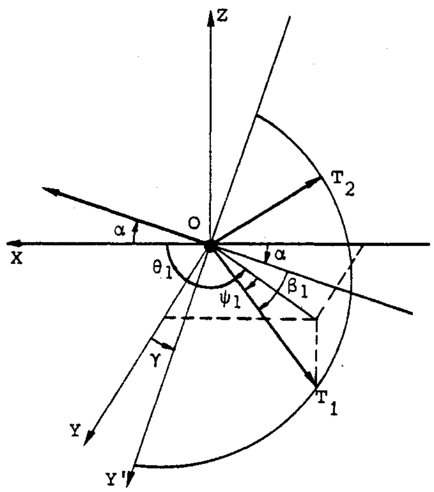

Fig. 6 Various rotations.

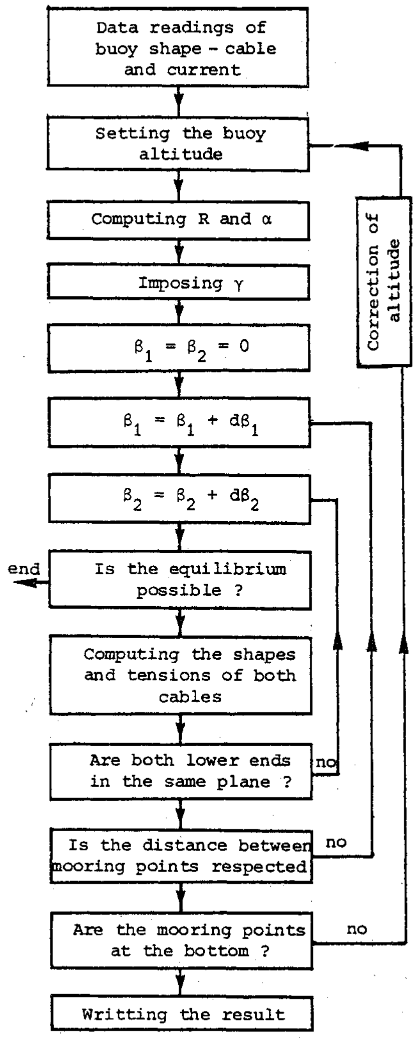

Fig. 7 Computer set.

$$tg\psi_l = \frac{T_{z_l}}{\sqrt{T_{x_l}^2 + T_{y_2}^2}} \tag{22}$$

Similar equations are assumed for the second cable.

Solutions

As one knows, the buoy and current characteristics R and α are computed given a plausible value to the "buoy altitude."

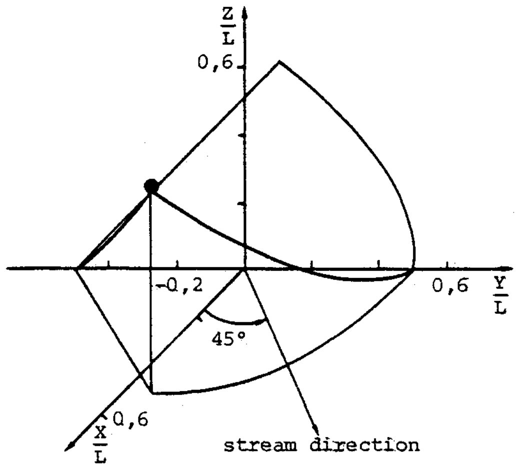

Fig. 8 Outlook of an entire mooring system.

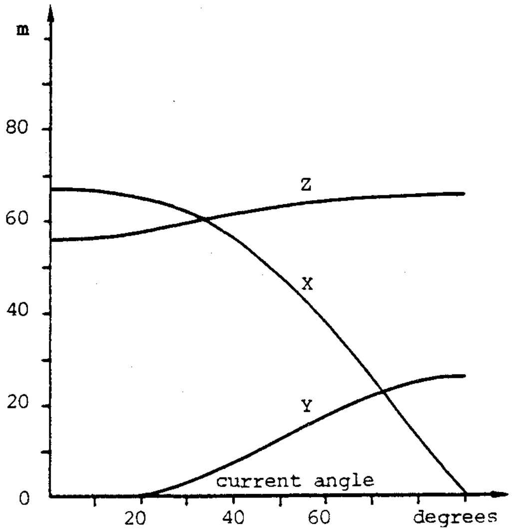

Fig. 9 Buoy coordinates.

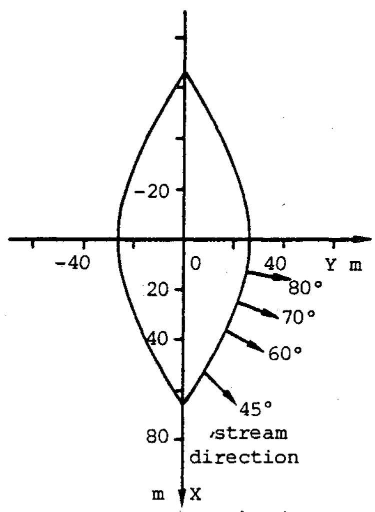

Fig. 10 Buoy displacements.

The buoy equilibrium conditions do not completely determine both cable tensions [we have to compute γ, β_1, β_2, T_1, and T_2 with only Eqs. (15) and (16)]. Therefore, we a priori select three values γ, β_1, and β_2 to determine T_1 and T_2. The cable shapes are then computed and the spacing of the mooring points is compared to the desired one.

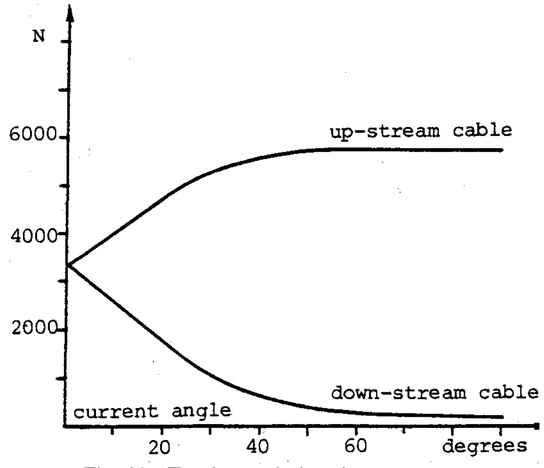

Fig. 11 Tension variations in the cables.

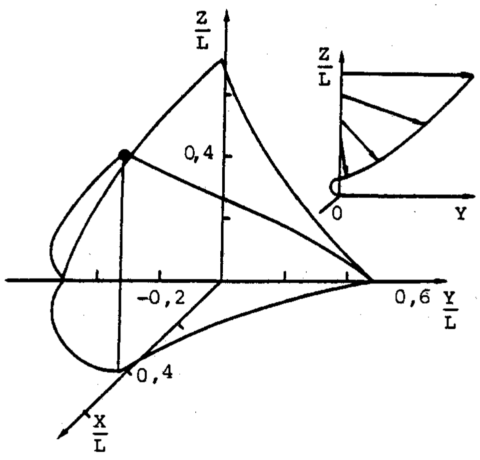

Fig. 13 Mooring in strengths and direction-varying current.

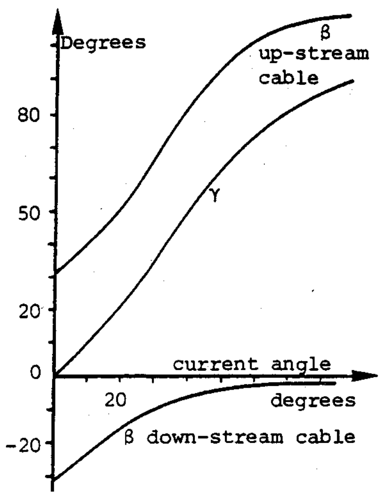

Fig. 12 Characteristic angles.

The problem is solved by means of successive iterations relating to buoy altitude, plane T_1, T_2 orientation γ, and angles β_1 and β_2. The following computer set, Fig. 7, gives the solving method and the various tests that are necessary. Note that γ is related to the current orientation with respect to the line joining the mooring points. A γ scanning will give every possible configuration.

Figures 8-13 show the results for a systematic analysis for some particular conditions.

A submarine buoy whose buoyancy is 4000 N has to be moored. The drag force in a 1 m/s current is 1000 N. Two identical 100-m long cables are used. When they are subjected to a uniform perpendicular current of 1 m/s, their drag force per unit length is 2 N, their friction coefficient is equal to 0.02, and their apparent weight is 1.15 N/m. The depth, as well as the distance between the mooring points, is 100 m. The current magnitude is a constant 2 m/s. The variations of the buoy position and the cable tensions are studied vs the current direction.

Each cable is divided into 25 elements and tests are within 1/100th of the desired mooring distance.

Figure 8 presents an outlook of an entire mooring system. Figure 9 gives the buoy coordinate variations with respect to the current angle. Let us note that its altitude does not change very much. Figure 10 presents a projection onto the horizontal plane of the buoy "displacement" with respect to the current angle. Figure 11 indicates the tension variations in the cable at the buoy. Finally, Fig. 12 displays the values of β_1, β_2, and γ vs current angle. This entire computation has only been made for a uniform current in order to make the results easier to understand. Figure 13 is the same mooring configuration, but in a strength and direction varying current:

$$V = 2x[Z/H + (Z/H)^3 - (Z/H)^4] \text{m/s} \tag{23}$$

$$\varphi = 90Z/H \text{ deg} \tag{24}$$

Conclusion

It seems to us that these examples show the great capabilities of our computation method for determining shapes and tensions of submarine cables. We suggest that this is a way to solve a number of general static cable problems. In particular, this method could be used to solve the moorings of floating bodies, determining, through new iteration, the cable length needed to let the buoy reach the free surface. Finally, one can note that the relative simplicity of the equations allows the possibility of using small computers (possibly a microcomputer) for solving these problems.

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